

RESEARCH ARTICLE

Short Introduction to SuperHyperGraph Theory with Some Applications

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ABSTRACT

A finite *hypergraph* extends an ordinary graph by permitting each *hyperedge* to connect an arbitrary nonempty subset of vertices, thereby encoding genuinely multiway interactions. Building on this idea, a finite *SuperHyperGraph* is formed by iterating the powerset construction, so that set-valued objects created at one level can act as vertices (and hence as potential edge endpoints) at the next. This provides a principled framework for representing hierarchical, nested, and multi-layer relational structures. In this paper, we introduce SuperHyperGraphs and several related concepts, illustrating them with concrete examples.

Keywords: Recursive SuperHyperGraph; HyperGraph; SuperHyperGraph; Hierarchical SuperHyperGraph.

Mathematics Subject Classification: 05C65 (MSC2010).

1 SuperHyperGraphs

We first review the set-theoretic constructions underlying SuperHyperGraphs and then recall the associated graph-theoretic terminology [1–4].

Definition 1.1 (Hypergraph). [5, 6] A *hypergraph* is an ordered pair $H = (V, E)$ where

- V is a finite set of *vertices*, and
- E is a finite family of nonempty subsets of V , called *hyperedges*.

Thus a hyperedge may involve more than two vertices, allowing one to encode genuinely multiway relations.

Definition 1.2 (n -fold iterated powerset). [7, 8] Let X be a set. Define $\mathcal{P}^1(X) := \mathcal{P}(X)$ and, for $n \geq 1$, recursively set

$$\mathcal{P}^{n+1}(X) := \mathcal{P}(\mathcal{P}^n(X)).$$

When the empty set is excluded, we write

$$\mathcal{P}_*^n(X) := \mathcal{P}^n(X) \setminus \{\emptyset\}.$$

Definition 1.3 (n -SuperHyperGraph). (see [1]) Let V_0 be a finite, nonempty base set, and define iterated powersets by

$$\begin{aligned} \mathcal{P}^0(V_0) &:= V_0, \\ \mathcal{P}^{k+1}(V_0) &:= \mathcal{P}(\mathcal{P}^k(V_0)) \quad (k \in \mathbb{N} \cup \{0\}). \end{aligned}$$

For $n \geq 0$, an n -SuperHyperGraph on V_0 is a pair

$$\text{SHG}^{(n)} = (V, E)$$

such that

$$V \subseteq \mathcal{P}^n(V_0) \quad \text{and} \quad E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

The elements of V are called *n-supervertices*, and the elements of E are called *superhyperedges*; equivalently, each superhyperedge is a nonempty subset of the supervertex set V .

As a brief comparison, Table 1 summarizes the main differences between a Hypergraph and an *n*-SuperHyperGraph. Compared with a Hypergraph, an *n*-SuperHyperGraph captures nested, hierarchical, and multi-level relations by allowing set-based vertices, thereby modeling richer higher-order structures and interactions in practice. In fact, SuperHyperGraphs have recently been discussed in a wide variety of publications [9–13].

Table 1. Comparison of Hypergraph and *n*-SuperHyperGraph

Aspect	Hypergraph	<i>n</i> -SuperHyperGraph
Basic form	$H = (V, E)$	$\text{SHG}^{(n)} = (V, E)$
Vertex set	V is a finite set of ordinary vertices	$V \subseteq \mathcal{P}^n(V_0)$, so vertices are <i>n</i> -th level set-based objects
Edge set	E is a family of nonempty subsets of V	$E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$, so each superhyperedge is a nonempty subset of the supervertex set
Structural level	Single-level structure	Multi-level or hierarchical structure determined by the iterated powerset level <i>n</i>
Underlying objects	Vertices are atomic objects	Supervertices may themselves encode nested collections of lower-level objects
Main purpose	Models multiway relations among ordinary vertices	Models multiway relations among higher-order, nested, or hierarchical entities
Set-theoretic basis	Uses only subsets of the vertex set	Uses iterated powersets of a base set V_0
Special case	Standard model	For $n = 0$, one obtains the ordinary hypergraph-type setting on a subset of V_0

A concrete example of a SuperHyperGraph is given below.

Example 1.4 (Multi-level product bundles in an e-commerce platform). Consider an e-commerce platform that offers individual products, simple bundles, and higher-level bundle packages.

Base set. Let

$$V_0 = \{\text{Laptop, Mouse, Dock, Monitor}\}.$$

Level-1 bundles. Define

$$\begin{aligned} b_1 &:= \{\text{Laptop, Mouse}\}, & b_2 &:= \{\text{Dock}\}, \\ b_3 &:= \{\text{Laptop, Dock}\}, & b_4 &:= \{\text{Monitor}\}. \end{aligned}$$

Choose $n = 2$. A 2-supervertex is a set of bundles. Define

$$v_1 := \{b_1, b_2\}, \quad v_2 := \{b_3, b_4\}.$$

Here, v_1 represents a basic work package, while v_2 represents an extended desk package.

Let

$$V := \{v_1, v_2\} \subseteq \mathcal{P}^2(V_0).$$

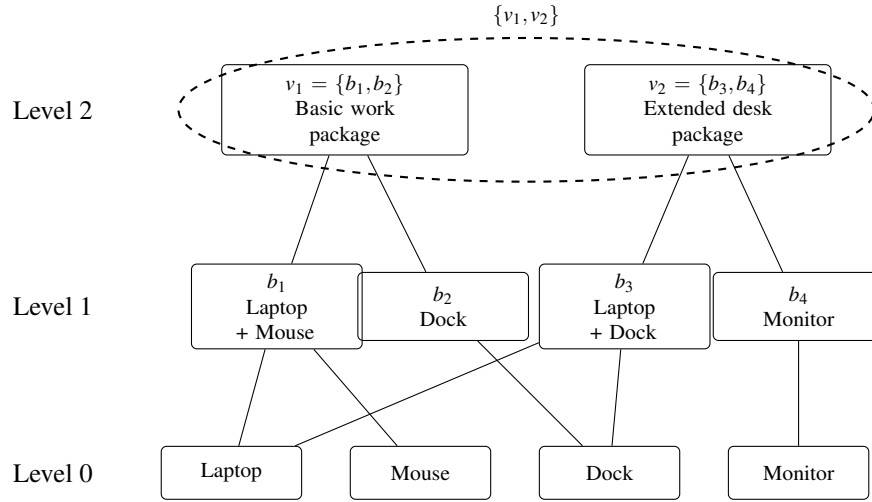


Figure 1. A simple 2-SuperHyperGraph for multi-level product bundles in an e-commerce platform.

Superhyperedge. Define

$$E := \{\{v_1, v_2\}\} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

This superhyperedge models a promotion that jointly recommends both higher-level packages.

Thus

$$\text{SHG}^{(2)} = (V, E)$$

is a concrete 2-SuperHyperGraph describing multi-level product bundling in e-commerce.

2 (m, n) -SuperHyperGraph

An (m, n) -SuperHyperGraph encodes multiway relations among supervertices modeled as maps $\mathcal{P}^m(S) \rightarrow \mathcal{P}^n(S)$; its edges are nonempty collections of such maps [2].

Definition 2.1 ((m, n) -SuperHyperGraph). Fix $m, n \in \mathbb{N}$ and a nonempty base set S . Let

$$\mathfrak{F}_{m,n}(S) := \{f : \mathcal{P}^m(S) \rightarrow \mathcal{P}^n(S)\}$$

be the class of all (m, n) -superhyperfunctions on S , where $\mathcal{P}^t$ denotes the t -fold iterated powerset. An (m, n) -SuperHyperGraph over S is a triple

$$\text{SHG}^{(m,n)} = (V, \mathcal{E}, \partial),$$

where:

- (i) $V \subseteq \mathfrak{F}_{m,n}(S)$ is a nonempty set (its elements are called (m, n) -supervertices);
- (ii) $\mathcal{E}$ is a nonempty set of (super)edge identifiers;
- (iii) $\partial : \mathcal{E} \rightarrow \mathcal{P}^*(V)$ is an incidence map, where $\mathcal{P}^*(V) := \mathcal{P}(V) \setminus \{\emptyset\}$.

For $e \in \mathcal{E}$, the set $\partial(e) \subseteq V$ is called the *incidence set* (or endpoint set) of the superhyperedge e .

Example 2.2 (Emergency-department protocol coordination). Consider an emergency department in which clinicians combine multiple clinical protocols to decide *bundles* of actions from observed findings.

Base set. Let the ground set S contain atomic findings and atomic actions:

$$S = \{F, O, C, B, T, X, K\},$$

where F = fever, O = low oxygen saturation, C = chest pain, B = order blood test, T = order CT imaging, X = start oxygen therapy, K = call cardiology.

Choose $(m, n) = (1, 2)$. Then a supervertex is a function

$$f : \mathcal{P}(S) \longrightarrow \mathcal{P}^2(S) = \mathcal{P}(\mathcal{P}(S)),$$

mapping a set of observed findings (and any already-selected items) to a *family of action-bundles* (each bundle is a subset of S).

Supervertices (protocols). Define three $(1, 2)$ -superhyperfunctions:

$$f_{\text{resp}}(A) := \begin{cases} \{\{X, B\}, \{X, T\}\}, & \text{if } O \in A, \\ \emptyset, & \text{otherwise,} \end{cases}$$

$$f_{\text{card}}(A) := \begin{cases} \{\{K, B\}, \{K, T\}\}, & \text{if } C \in A, \\ \emptyset, & \text{otherwise,} \end{cases}$$

$$f_{\text{inf}}(A) := \begin{cases} \{\{B\}, \{B, X\}\}, & \text{if } F \in A, \\ \emptyset, & \text{otherwise.} \end{cases}$$

Let

$$V := \{f_{\text{resp}}, f_{\text{card}}, f_{\text{inf}}\} \subseteq \mathfrak{F}_{1,2}(S).$$

Superhyperedges (joint-use situations). Let the edge-identifier set be

$$\mathcal{E} := \{e_{\text{pne}}, e_{\text{acs}}, e_{\text{complex}}\},$$

with incidence map $\partial : \mathcal{E} \rightarrow \mathcal{P}^*(V)$ defined by

$$\partial(e_{\text{pne}}) = \{f_{\text{resp}}, f_{\text{inf}}\}, \quad \partial(e_{\text{acs}}) = \{f_{\text{resp}}, f_{\text{card}}\}, \quad \partial(e_{\text{complex}}) = \{f_{\text{resp}}, f_{\text{card}}, f_{\text{inf}}\}.$$

Interpretation: each superhyperedge collects the protocols that must be considered together in a given clinical scenario (e.g., pneumonia-like, acute-coronary-like, or a complex case). Thus $\text{SHG}^{(1,2)} = (V, \mathcal{E}, \partial)$ is a concrete (m, n) -SuperHyperGraph arising in real-world decision support.

3 Hierarchical Undirected SuperHyperGraphs

A hierarchical superhypergraph allows vertices drawn from several iterated-powerset levels and permits *mixed-level* edges, while imposing a coherence requirement that guarantees level-to-level consistency.

Definition 3.1 (Hierarchical SuperHyperGraph of height r). Let V_0 be a finite, nonempty base set and fix $r \in \mathbb{N}_0$. Define the *nonempty powerset tower* $(\mathcal{P}^{(k)}(V_0))_{k=0}^r$ by

$$\mathcal{P}^{(0)}(V_0) := V_0, \quad \mathcal{P}^{(k+1)}(V_0) := \mathcal{P}(\mathcal{P}^{(k)}(V_0)) \setminus \{\emptyset\} \quad (0 \leq k < r).$$

Set the *hierarchical universe* to be

$$\mathcal{U}_r(V_0) := \bigcup_{k=0}^r \mathcal{P}^{(k)}(V_0).$$

For $x \in \mathcal{U}_r(V_0)$, define its *level* by

$$\ell(x) := \min\{k \in \{0, 1, \dots, r\} : x \in \mathcal{P}^{(k)}(V_0)\}.$$

A *hierarchical SuperHyperGraph of height r* on V_0 is a pair

$$\mathbb{H}^{(r)} = (V, E)$$

satisfying the following conditions:

(H1) (*Hierarchical vertex set*). V is a finite, nonempty set such that

$$V \subseteq \mathcal{U}_r(V_0).$$

The elements of V are called *hierarchical supervertices*.

(H2) (*Cross-level edges*). E is a finite family of nonempty subsets of V , that is,

$$E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

The elements of E are called *hierarchical superhyperedges*. In particular, a single edge may contain vertices from different levels.

(H3) (*Coherence / downward closure*). Whenever $X \in V$ has level at least 1, all of its immediate constituents are also required to be vertices:

$$X \subseteq V.$$

For each $k \in \{0, \dots, r\}$, the k -th layer is defined by

$$V_k := \{x \in V : \ell(x) = k\}, \quad \text{so that} \quad V = \bigcup_{k=0}^r V_k.$$

Example 3.2 (Hierarchical sensor monitoring as a height-2 hierarchical SuperHyperGraph). Consider a building-wide monitoring system.

Ground level (V_0). Let the base set be the sensors installed in the building:

$$V_0 = \{S_1, S_2, S_3, S_4\},$$

where each S_i is a physical sensor node.

Height $r = 2$ (three levels: 0, 1, 2). Level-0 vertices represent individual sensors. Level-1 vertices represent *nonempty sensor clusters* (e.g., devices within a room or corridor segment). Level-2 vertices represent *nonempty collections of clusters* (e.g., zones composed of multiple rooms).

Hierarchical supervertex set. Define the following mixed-level vertices:

$$\begin{aligned} V'_0 &:= \{S_1, S_2, S_3, S_4\}, \\ c_1 &:= \{S_1, S_2\}, \quad c_2 := \{S_2, S_3\}, \quad c_3 := \{S_3, S_4\}, \\ z_1 &:= \{c_1, c_2\}, \quad z_2 := \{c_2, c_3\}. \end{aligned}$$

Let

$$V := V'_0 \cup \{c_1, c_2, c_3\} \cup \{z_1, z_2\} \subseteq \mathcal{U}_2(V_0).$$

Interpretation: c_i are local clusters of nearby sensors, while z_1, z_2 are higher-level zones formed from clusters.

Downward-closure (coherence). Since $c_1, c_2, c_3 \in V$ have level 1, their constituents S_i are also in V . Since $z_1, z_2 \in V$ have level 2, their constituents c_i are also in V . Hence the coherence requirement $X \subseteq V$ for all $X \in V$ with $\ell(X) \geq 1$ is satisfied.

Cross-level edges. Define a set of hierarchical superhyperedges by

$$E := \{\{S_2, c_2, z_1\}, \{c_2, c_3, z_2\}, \{S_1, c_1\}\} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

Interpretation: the edge $\{S_2, c_2, z_1\}$ links an individual sensor, its local cluster, and a higher-level zone, capturing an alarm escalation path from device-level signals to cluster-level and zone-level aggregation.

Therefore $\mathbb{H}^{(2)} = (V, E)$ is a concrete hierarchical SuperHyperGraph of height $r = 2$ arising from real-world multi-level sensor monitoring. Figure 2 illustrates the mixed-level vertices, the containment relations required by downward closure, and the cross-level hierarchical superhyperedges.

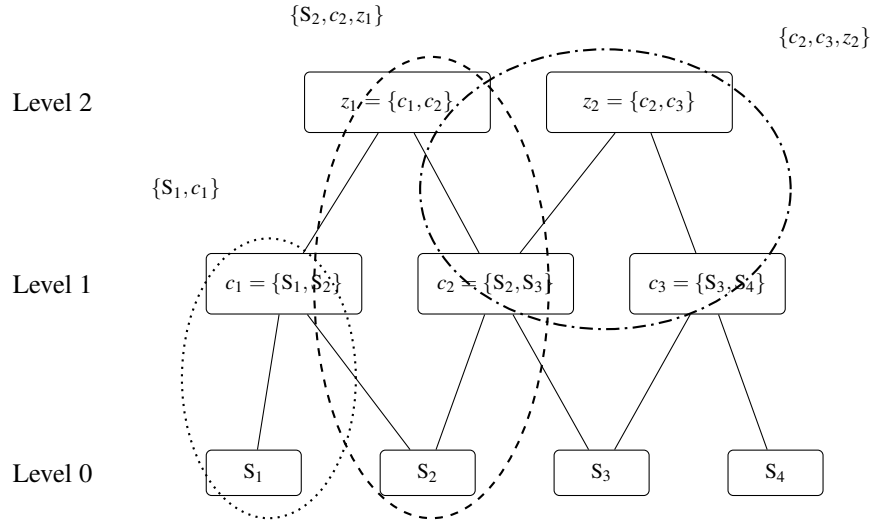


Figure 2. A height-2 hierarchical SuperHyperGraph for sensor monitoring. Straight segments indicate downward-closure/containment relations across levels, while the dotted and dashed closed curves represent cross-level hierarchical superhyperedges.

4 Recursive HyperGraph

A *Recursive HyperGraph* is a hypergraph in which hyperedges are allowed to contain not only ordinary vertices but also lower-level hyperedges as elements, thereby supporting nested (self-referential) incidence up to a bounded recursion depth [14, 15]. An (n, k) -recursive SuperHyperGraph combines level- n supervertices (via iterated powersets) with depth- k recursive superhyperedges that may contain supervertices and lower-level edges as elements [16, 17].

Definition 4.1 (Depth- k powerset universe). [14, 15] Let S be a nonempty set and let $k \in \mathbb{N} \cup \{0\}$. Define a hierarchy of sets $(S_i)_{i \geq 0}$ by

$$S_0 := S, \quad S_i := \mathcal{P}\left(\bigcup_{j=0}^{i-1} S_j\right) \quad (i \geq 1).$$

The *depth- k powerset universe* over S is

$$2_{S,k} := \mathcal{P}\left(\bigcup_{i=0}^k S_i\right).$$

Definition 4.2 (k -recursive hypergraph). [14, 15] Let V be a finite vertex set and let $k \in \mathbb{N} \cup \{0\}$. A *k -recursive hypergraph* is a pair

$$H = (V, E)$$

such that

$$E \subseteq 2_{V,k} \setminus \{\emptyset\},$$

where $2_{V,k}$ is the depth- k powerset universe from Definition 4.1 applied to $S = V$.

In particular, when $k = 0$ one has $2_{V,0} = \mathcal{P}(V)$ and therefore $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$, so H reduces to an ordinary hypergraph.

Definition 4.3 ((n, k) -recursive SuperHyperGraph). [17] Fix a base (ground) set V_0 and let $n, k \in \mathbb{N} \cup \{0\}$. (*Iterated powersets*). Define the iterated powersets by

$$\mathcal{P}^0(V_0) = V_0, \quad \mathcal{P}^{n+1}(V_0) = \mathcal{P}(\mathcal{P}^n(V_0)) \quad (n \geq 0).$$

A (n, k) -recursive SuperHyperGraph is a pair

$$\text{RSHG}^{(n,k)} = (V, E)$$

satisfying:

- (i) (Hierarchical supervertex set). $V \subseteq \mathcal{P}^n(V_0)$.
- (ii) (Recursive superhyperedge family). $E \subseteq 2_{V,k} \setminus \{\emptyset\}$, where $2_{V,k}$ is the depth- k powerset universe constructed from $S = V$ as in Definition 4.1.

Example 4.4 (Hierarchical incident response in a cloud service). Consider a cloud-based microservice system.

Ground level. Let V_0 be the set of basic components (services and on-call teams), e.g.,

$$V_0 = \{\text{Auth}, \text{API}, \text{DB}, \text{Cache}, \text{SRE}, \text{Sec}\}.$$

Choose $n = 2$ (supervertices as “sets of component-groups”). A level-2 supervertex is an element of $\mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}(V_0))$, so it represents a collection of component groups. For instance, define

$$v_1 := \{\{\text{Auth}, \text{API}\}, \{\text{DB}\}\}, \quad v_2 := \{\{\text{Cache}\}, \{\text{SRE}\}\},$$

$$v_3 := \{\{\text{Sec}\}, \{\text{API}, \text{DB}\}\}.$$

Interpretation: v_1 encodes the application-and-database dependency group, v_2 encodes the caching layer with the SRE team, and v_3 encodes a security-related incident involving the API/DB boundary.

Let the supervertex set be

$$V := \{v_1, v_2, v_3\} \subseteq \mathcal{P}^2(V_0).$$

Choose $k = 1$ (recursive superhyperedges may include supervertices and lower-level edges). For $k = 1$, the depth-1 universe $2_{V,1}$ contains subsets of V as well as subsets whose elements may themselves be subsets of V (i.e., “edge-of-edges” structures).

Define two level-0 incident bundles (ordinary hyperedges on V):

$$e_1 := \{v_1, v_2\} \subseteq V, \quad e_2 := \{v_2, v_3\} \subseteq V.$$

Interpretation: e_1 models an incident where the application/database group and caching/SRE group must coordinate; e_2 models an incident where caching/SRE and security/API-DB boundary must coordinate.

Now define a recursive superhyperedge that bundles these incident bundles:

$$e_3 := \{e_1, e_2\} \in \mathcal{P}(\mathcal{P}(V)) \subseteq 2_{V,1}.$$

Interpretation: e_3 represents a higher-level escalation (e.g., a major outage) whose resolution requires handling both incident bundles e_1 and e_2 together, capturing a nested coordination structure.

Finally, set

$$E := \{e_1, e_2, e_3\} \subseteq 2_{V,1} \setminus \{\emptyset\}.$$

Then $\text{RSHG}^{(2,1)} = (V, E)$ is a concrete $(n, k) = (2, 1)$ -recursive SuperHyperGraph modeling real-world hierarchical incident management with nested escalation bundles. Figure 3 visualizes the base components, the intermediate component-groups, the level-2 supervertices, the ordinary incident bundles e_1, e_2 , and the recursive superhyperedge $e_3 = \{e_1, e_2\}$.

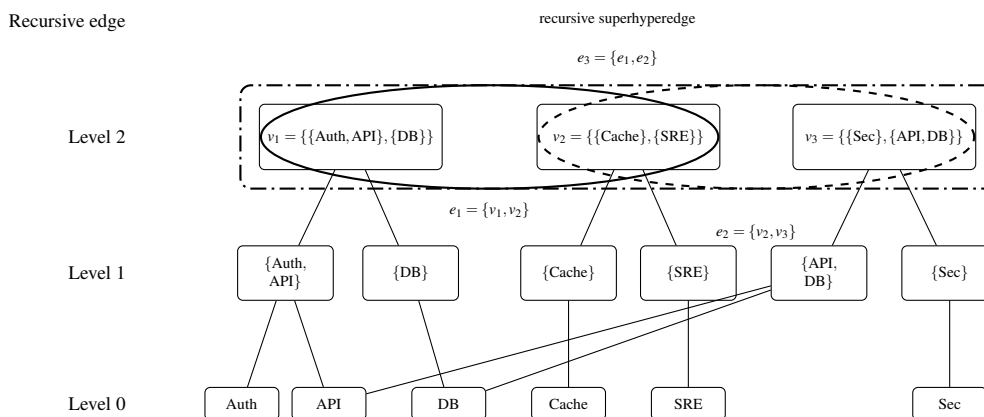


Figure 3. A concrete $(2, 1)$ -recursive SuperHyperGraph for hierarchical incident response in a cloud service. Thin segments indicate containment relations across levels, the solid and dashed closed curves represent the ordinary incident bundles e_1 and e_2 , and the outer dash-pattern box represents the recursive superhyperedge $e_3 = \{e_1, e_2\}$.

5 Open Problems

The SuperHyperGraph frameworks introduced here raise several natural research questions.

- (i) **Algorithmic design.** Can one develop efficient algorithms for SuperHyperGraphs and their variants (e.g., construction, traversal, matching, covering, domination, coloring, optimization, and recognition problems), together with complexity and approximation guarantees?
- (ii) **Neural-network applications.** Can SuperHyperGraphs be used as underlying data structures for neural models that learn from hierarchical and multiway relations, for instance by extending message-passing and attention mechanisms to iterated-powerset vertex levels? (cf. [18, 19])
- (iii) **Graph parameters and invariants.** Which classical graph/hypergraph parameters admit meaningful analogues in the SuperHyperGraph setting (e.g., degree notions, connectivity, acyclicity, width measures, spectral quantities, domination/covering numbers), and how do these invariants behave under level changes and natural SuperHyperGraph operations?

Declarations

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Conflicts of interest. The author declares that he has no competing interests.

Ethics approval. Not applicable.

Data availability. No data were generated or analysed during this study.

Author contributions. The author is solely responsible for the conceptualization, analysis, and writing of this manuscript.

Use of artificial intelligence. We use generative AI and AI-assisted tools for tasks such as English grammar checking, and we do not employ them in any way that violates ethical standards.

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